Limits on active-sterile neutrino mixing and the primordial deuterium abundance

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Studies of limits on active-sterile neutrino mixing derived from big bang nucleosynthesis (BBN) considerations are extended to consider the dependence of these constraints on the primordial deuterium abundance. This study is motivated by recent measurements of D/H in quasar absorption systems, which at present yield discordant results. Limits on active-sterile mixing are somewhat relaxed for high D/H (≈2×10^{-5}). For low D/H (≈2×10^{-8}), no active-sterile neutrino mixing is allowed by currently popular upper limits on the primordial 4He abundance Y. For such low primordial D/H values, the observational inference of active-sterile neutrino mixing by upcoming solar neutrino experiments would imply that Y has been systematically underestimated, unless there is new physics not included in standard BBN.

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Upper limits on the abundance of 4He produced in big bang nucleosynthesis (BBN) have been used to limit mixing between active (νe, νμ, or ντ) and sterile (νs, no standard model interactions) neutrinos [1]. In this paper, we point out and discuss how these constraints are dependant on the adopted primordial deuterium abundance. Previous limits on sterile neutrino mixing have assumed a value for the lower bound on the baryon-to-photon ratio η derived from interstellar medium and solar system measurements of deuterium (D) and 3He, and models of chemical and galactic evolution. Recent measurements of D/H in quasar absorption systems (QASs) have yielded discordant values of this ratio, some higher than previously derived ranges [2], and some lower [3]. Several factors make an investigation of the primordial D/H dependence of BBN constraints on active-sterile neutrino mixing timely: the discordant QAS measurements of D/H; the fact that future solar neutrino experiments may be able to distinguish and identify νe-νs mixing [4]; and the use of sterile neutrinos in schemes for neutrino masses and mixings that explain all available data [5].

As is well known (e.g., Ref. [6]), the abundance of 4He produced by BBN is essentially determined by the ratio of neutron to proton number densities (n/p) at “weak freeze-out” (WFO). WFO occurs when the reactions that interchange neutrons and protons proceed too slowly relative to the expansion rate of the universe to keep n/p at its equilibrium value of n/p=exp(-Δm/T). Here Δm=m_n−m_p=1.293 MeV is the neutron-proton mass difference, and T is the photon temperature. Mixing between active and sterile neutrinos increases (n/p)WFO, and therefore the primordial 4He mass fraction Y, in two ways. First, active-sterile neutrino mixing effectively brings more degrees of freedom into thermal contact, increasing the energy density and hence the expansion rate of the universe. Second, active-sterile mixing—especially νe-νs mixing—depletes the electron neutrino and antineutrino populations, reducing the rates of the n→p interconversion reactions. Both of these effects cause n/p to freeze out at a higher temperature.

Using a neutrino ensemble evolution formalism [1,7] that includes both neutrino oscillations (with matter effects) and neutrino collisions, previous authors [1] have produced exclusion plots in the δm^2-sin^22θ plane for both νe-νs and νμ-νs mixing [8]. Here δm^2 and sin^22θ are the difference of the squares of the neutrino vacuum mass eigenvalues and a measure of the vacuum mixing angle, respectively, associated with two-flavor neutrino mixing. These studies showed that for η>2.8×10^{-10}, both the νμ-νs solution to the atmospheric neutrino problem [9] and the νe-νs large-angle Mikheyev-Smirnov-Wolfenstein (MSW) solution to the solar neutrino problem [10] are excluded for Y<0.247.

In our study of the primordial D/H dependence of BBN constraints on active-sterile neutrino mixing, we have employed the same neutrino evolution formalism [1,7] as previous authors. We have neglected any net lepton number contributed by the neutrinos. (An initial lepton number of greater than about 10^{-4}—about six orders of magnitude larger than the known net baryon number—would allow the limits presented here to be evaded [11]. Also, the recently reported effect of active-sterile neutrino mixing generating a net lepton number does not occur in the regions of parameter space we consider here [12].) In this case, the neutrino and antineutrino sectors evolve identically. A Fermi-Dirac momentum distribution for all neutrinos is assumed, but allowance is made for nonequilibrium number densities. The differential equations in the formalism yield n_nu,n_nubar,n_nu,n_nubar, and T as functions of time. Here n_nu denotes the fraction of a full fermionic degree of freedom contributed by neutrino species x, which we shall hereafter call the “number density parameter” of neutrino species x. In the equations below we will take n_nu=n_nubar, since we are working under the assumption that the net lepton number contributed by the neutrinos is negligible.

In our BBN computation we have employed the Kawano [13] update of the Wagoner [14] code, with the latest world average neutron lifetime τ≈887.0 s [15], the reaction rates of Ref. [16], and a correction of +0.0031 to Y due to finite nucleon mass and timestep-dependent effects [17]. We have altered the Kawano code to use the ‘temperature series’ of neutrino number density parameters n_nu(T) to compute the energy density contributed by neutrinos and the n→p interconversion rates. The neutrino energy density is

\[ \rho_{\nu} = \frac{7}{8} \frac{\pi^2}{15} (n_{\nu_e} + n_{\nu_\mu} + n_{\nu_\tau} + n_{\nu_s}) T^4. \] (1)

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The \( n \leftrightarrow p \) rates are

\[
\lambda_{n \to p} = K \int_1^\infty \frac{1}{1+e^{xz}} \left( 1 - \frac{n_{\nu_e}}{1+e^{(x+q)z}} \right) x(x+q)^2(x^2-1)^{1/2} dx,
\]

\[
\lambda_{p \to e} = K \int_q^\infty \frac{1}{1+e^{xz}} \left( 1 - \frac{n_{\nu_e}}{1+e^{(x-q)z}} \right) x(x-q)^2(x^2-1)^{1/2} dx,
\]

\[
\lambda_{n \to e} = K \int_q^\infty \frac{1}{1+e^{xz}} \left( 1 - \frac{n_{\nu_e}}{1+e^{(x-q)z}} \right) x(x+q)^2(x^2-1)^{1/2} dx,
\]

\[
\lambda_{p \to e} = K \int_q^\infty \frac{1}{1+e^{xz}} \left( 1 - \frac{n_{\nu_e}}{1+e^{(x+q)z}} \right) x(x-q)^2(x^2-1)^{1/2} dx,
\]

\[
\lambda_{p \to n} = K \int_q^\infty \frac{1}{1+e^{xz}} \left( 1 - \frac{n_{\nu_e}}{1+e^{(x+q)z}} \right) x(x+q)^2(x^2-1)^{1/2} dx,
\]

\[
\lambda_{n \to e} = K \int_q^\infty \frac{1}{1+e^{xz}} \left( 1 - \frac{n_{\nu_e}}{1+e^{(x-q)z}} \right) x(x-q)^2(x^2-1)^{1/2} dx.
\]

In these expressions, \( x=E_e/m_e \), where \( E_e \) and \( m_e \) are the total electron (or positron) energy and rest mass, respectively; \( z=m_e/T \); \( z_{\nu_e}=m_{\nu_e}/T_{\nu_e} \), where \( T_{\nu_e} \) is the appropriate neutrino temperature; \( q=\Delta m/m_{\nu_e} \); and \( K \) is a constant obtained by solving the equation

\[
\lim_{z, z_{\nu_e} \to \infty} \lambda_{n \to p} = 1/\tau
\]

for \( K \), where \( \tau \) is the experimentally measured neutron lifetime.

The lower limit on \( \eta \) obtained from a standard BBN calculation with \( N_{\nu_e}=3 \) is not appropriate for BBN with active-sterile mixing. This is because the lower limit on \( \eta \) depends on the expansion rate, often codified as an effective number of neutrino generations \( N_{\nu} \) [18]. Since active-sterile mixing increases \( N_{\nu} \) (at least for the range of parameter space we consider here [12]), it affects the lower bound on \( \eta \). Therefore, we will plot our results as a function of the primordial D/H value—the experimentally determined quantity—rather than as a function of \( \eta \). These considerations are most important for the \( \nu_{\mu} \leftrightarrow \nu_{\tau} \) atmospheric neutrino mixing solution, and much less important (nearly negligible) for the \( \nu_{\mu} \leftrightarrow \nu_{s} \) small-angle MSW solution to the solar neutrino problem.

In Fig. 1, a representative \( \nu_{\mu} \leftrightarrow \nu_{s} \) atmospheric neutrino mixing solution (\( \Delta m^2 = 1.0 \times 10^{-2} \text{ eV}^2 \), \( \sin^2 2\theta = 0.6 \)) is assumed, and the resulting BBN \(^4\text{He} \) yield is plotted as a function of the BBN D/H yield. The value of \( \eta \) (given as \( \eta_{10} = 10^{10} \eta \)) at various values of D/H is also indicated on the figure. For a given value of D/H, the implied abundance of \(^4\text{He} \) can be interpreted as the observational upper limit required to constrain the solution. Alternatively, a detection of these neutrino mixing parameters by, for example, future astrophysical neutrino experiments would yield an independent determination of the primordial \(^4\text{He} \) abundance, so long as D/H were known from QAS studies. This could be very interesting, given the recent emphasis on the systematic uncertainties in the determination of the primordial \(^4\text{He} \) abundance as derived from helium recombination lines in extragalactic H II regions [19]. The conclusions reached from Fig. 1 are essentially the same over the range of \( \Delta m^2 \) \( (10^{-3} - 10^{-1} \text{ eV}^2) \) for the proposed \( \nu_{\mu} \leftrightarrow \nu_{s} \) mixing explanation.
\[ \nu_{\mu} - \nu_{\tau} \] atmospheric neutrino solution is still somewhat constrained [24] if current observational inferences [25] of primordial $^4$He are correct: $Y = 0.234 \pm 0.003 \pm 0.005$, where the first error is statistical and the second systematic. Of course, if this $\nu_{\mu} - \nu_{\tau}$ atmospheric neutrino mixing solution were inferred from atmospheric neutrino experiments, and QAS studies confirm D/H \( \approx 2 \times 10^{-4} \), the implied $^4$He abundance of $Y \approx 0.245$ would be significantly higher than the central value of $Y = 0.234$ cited above.

The $\nu_{\mu} - \nu_{\tau}$ small-angle MSW solar neutrino solution is allowed for D/H \( \approx 2 \times 10^{-4} \). Figure 2 shows that an observational upper bound of $Y \leq 0.232$ would be required to restrict this small angle solution if such a high D/H is indeed the primordial value.

Other very high quality QAS data—arguably better [3] for the determination of D/H than that used in Ref. [2]—suggest D/H \( \approx 2 \times 10^{-5} \) [3]. This value of D/H is incompatible with standard BBN with $N_e = 3$ [17,18,26] for current observational inferences of the primordial $^4$He abundances [25], and any mixing with sterile neutrinos would only exacerbate the problem. As mentioned previously, however, it has been argued that $Y$ has been systematically underestimated, and a more appropriate upper limit on $Y$ may actually be $Y \leq 0.255$ [19]. It is unlikely that the systematic error in $Y$ is enough to allow the $\nu_{\mu} - \nu_{\tau}$ atmospheric neutrino solution for D/H \( \approx 2 \times 10^{-5} \). However, observation of the $\nu_{\mu} - \nu_{\tau}$ small-angle MSW solar neutrino solution, together with a solid determination of D/H \( \approx 2 \times 10^{-4} \), would require that $Y$ has been systematically and significantly underestimated by about 0.015 (see Fig. 2), unless there is nonstandard physics during the BBN epoch [26]. This is a somewhat trivial point, since the mixing parameters of the small-angle $\nu_{\mu} - \nu_{\tau}$ MSW solution produce only slightly more $^4$He than the standard BBN picture with $N_e = 3$, for which the `crisis’ at low D/H is well known [17,26,18]. Useful constraints on the $\nu_{\mu} - \nu_{\tau}$ small-angle MSW solar neutrino solution would require very precise observational knowledge of $Y$ and $Y$. This may, however, still be interesting in view of the fact that future solar neutrino experiments may be able to distinguish the sterile neutrino oscillation-based solution from other solutions [4]. Also, many models that seek to satisfy all available constraints on neutrino properties employ the $\nu_{\mu} - \nu_{\tau}$ small-angle MSW solar neutrino solution [5] (but see Ref. [27]).

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[8] $\nu_e - \nu_x$ mixing would have the same effects on $Y$ as $\nu_\mu - \nu_\tau$ mixing, but this is of less interest since neither the solar nor atmospheric neutrino anomalies could be solved with $\nu_e - \nu_x$ mixing.


[22] Rugers and Hogan in Ref. [2].


[24] There is a small but nontrivial discrepancy between our results and the those of Shi *et al.* [1]. For $\eta = 2.8 \times 10^{-10}$, the atmospheric neutrino solution would imply $Y = 0.248$ according to Shi *et al.* [1], while we obtain $Y = 0.252$. The difference is due to our use of the $+0.0031$ correction to $Y$, and the fact that we numerically integrate the altered $n \rightarrow p$ reactions in our BBN calculation. Thus the constraints on the $\nu_{\mu} - \nu_x$ atmospheric neutrino solution are not as relaxed as might have been expected for $\text{D/H} \approx 2 \times 10^{-4}$.

